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PRINCIPAL PARAMETERS AND OPERATING INDICES FOR REVERSIBLE FOLLOW-UP SYSTEMS

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ABSTRACT. An engineering approach to the analysis of the principal parameters and operating indices for reversible follow-up systems is presented. Structurally symmetrical and structurally asymmetrical systems are discussed.

Many remote-controlled master-slave manipulators have been designed to include the property of active reflection of the forces developed by the slave member on the operator's shaft. These systems received the designation reversible follow-up systems (RFS) because they follow from the master side, as well as from the slave side [1]. 63

Reversible follow-up systems are a special class of two-dimensional automatic regulating systems, the control coordinates of which are the external moments generated by the operator and by the load. The control coordinates are considered to be the positions of the operator and load shafts.

The use of reversible follow-up systems to control master-slave manipulators fixes a number of particularly specific indices of their quality of performance. It is obvious that inherent in reversible follow-up systems as in any other automatic regulating system, are specific dynamic indices such as frequency properties and stability indices. Indices of quality of performance in transient modes are highly important in the designing of RFS.

However, at the basis of any method used to design RFS is the need to ensure first of all specific indices of quality of performance under steady-state conditions. This is so because the basic conditions under which master-slave manipulators perform, and in which the property of force reflection should be most completely manifested are just such steady-state conditions. These conditions refer to motion of master and slave shafts at constant speed under the influence of external moments generated by the operator and by the load. Moreover, the smaller the error in the measurement of the loading moments generated by the operator when the shafts move at a predetermined speed, the greater the sensitivity of the reversible system.

Moreover, when shafts are stationary, when the error in the operator's

perception of the loading moments is infinitesimally small, it is important to estimate the proportionality of the force reflection as well as the precision in matching the angular coordinates of the operator's shaft and the load shaft when external moments are acting. All these performance indices depend on definite reversible system parameters [2].

The literature does not, at this time, contain a general procedure for evaluating the principal reversible performance indices, and the relationship between system parameters and their performance indices under steady-state conditions is not always established. So it is difficult to make a comparative analysis of the different reversible system variants when designing master-slave manipulators and to make a valid selection of RFS parameters and elements for specified performance quality indices. This article attempts to fill this gap and suggests an engineering approach to analysis of RFS principal parameters and operating indices. As a practical matter, all types of reversible follow-up systems can be broken down into two basic types, which we shall arbitrarily call structurally symmetrical and structurally asymmetrical. /64

Let us understand a structurally symmetrical RFS to be one in which the main circuit consists of two symmetrical systems, closed with respect to position. The structurally symmetrical arrangement usually includes elements and couplings needed to compensate for the "internal" moments, and this causes asymmetry. However, this does not change the original structural symmetry of the system.

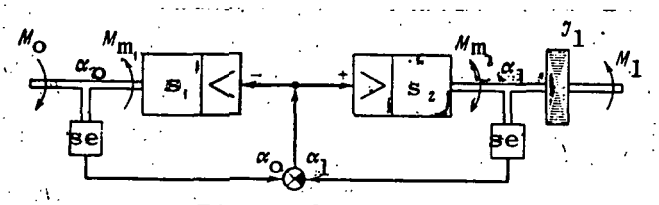

Figure 1.

A structurally asymmetrical RFS includes a slave system closed with respect to position, and a manipulator master element system closed with respect to moment. This enables the operator to control the position of the slave elements with the former and to perceive the force developed by the slave at the master. Let us move on to a consideration of these two basic types of RFS.

Let us examine the principal parameters and operating indices for a reversible follow-up system by using the example of the structurally-symmetrical system in Figure 1.

The dynamics of the operation of this system can be described by the following system of differential equations written in Laplace transforms for zero initial conditions

$$\begin{aligned} M_{m_1}(P) &= I_1 P^2 \alpha_o(P) - M_o(P); \\ M_{m_2}(P) &= (I_2 + I_1) P^2 \alpha_1(P) + M_1(P); \\ M_{m_1}(P) &= C_1(P)[\alpha_1(P) - \alpha_o(P)] - b_1(P) P\alpha_o(P); \\ M_{m_2}(P) &= C_2(P)[\alpha_o(P) - \alpha_1(P)] - b_2(P) P\alpha_1(P), \end{aligned} \tag{1}$$

where

$M_{m_1}(P)$, $M_{m_2}(P)$ are the moments developed by the slaves attributable to the operator, and load, respectively;

$M_o(P)$, $M_1(P)$ are the moments applied to the shafts of the slave elements by the operator and by the load;

$$I_1 = I_{m_1} + I_{k_1}; \quad I_2 = I_{m_2} + I_{k_2}, \tag{2}$$

where

I_m is the motor rotor moment of inertia;

I_k is the moment of inertia of the links in the kinematic chain for the corresponding degree of freedom;

I_1 is the moment of inertia of the load at the slave;

$\alpha_o(P)$, $\alpha_1(P)$ are the coordinates of operator and load shafts, respectively;

$C_1(P)$, $C_2(P)$ are the transfer functions of the elements in the system between the coordinates of the input and moment developed by the motors;

$b_1(P)$, $b_2(P)$ are the transfer functions of the internal negative feedbacks in terms of operator and load shaft speeds. /65

Thus, if nonself-excited direct current motors and armature circuit control are used as the slave elements in the system, and if the internal feedbacks used to correct system properties are lacking, we will have

(3)

where

k_m is the slave motor moment transmission coefficient;

R_a is the resistance of the motor armature circuit;

τ_e is the electrical time constant for the motor armature circuit;

k_ω is the motor's counter-emf coefficient;

$W_{am}(P)$ is the amplifier transfer function, including the series correction circuit.

Transforming the system of equations at (1), we obtain

$$\begin{aligned} M_o(P) &= M_1(P) + I_1 P^2 \alpha_o(P) + b_1(P) P \alpha_o(P); \\ M_2(P) &= M_1(P) + (I_2 + I_1) P^2 \alpha_1(P) + b_2(P) P \alpha_1(P); \\ M_1(P) &= C_1(P)[\alpha_o(P) - \alpha_1(P)]; \\ M_2(P) &= C_2(P)[\alpha_o(P) - \alpha_1(P)]. \end{aligned} \tag{4}$$

Figure 2 is a structural diagram corresponding to the systems of differential equations at (1) and (4).

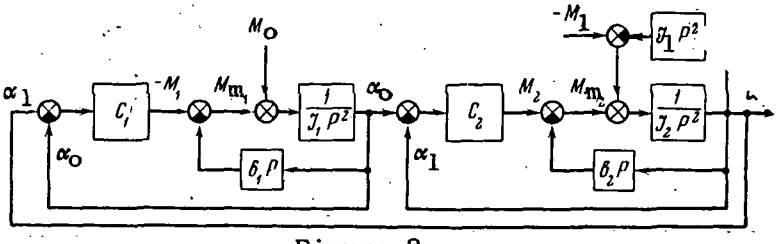

Figure 2.

As may be seen from the system of differential equations at (4), the precision with which position is followed by a reversible follow-up system is determined by the magnitude of the external moments, M_o and M_1, as well as by the parameters $C_1(P)$ and $C_2(P)$

(5)

$C_1(P)$ and $C_2(P)$ will be understood to be the magnitudes of the dynamic rigidity of the reversible follow-up system with respect to M_o and M_1.

In steady-state conditions, when M_o, M_1 = const; α_o, α_1 = const

$$\alpha_o - \alpha_1 = M_o / C_1 = M_1 / C_2, \tag{6}$$

where

C_1 and C_2 are the coefficients of the transfer functions $C_1(P)$, $C_2(P)$.

The error in the angular coordinates of the operator's shaft and of the load shaft is determined by the coefficients of the transfer functions for the elements in the circuit between the input to each drive, and the moments developed by the motors.

The experimental determination of these coefficients under steady-state conditions can be made by applying a moment that is constant in magnitude to the operator's shaft, or to the load shaft, with the position of one of the shafts locked. /66

(7)

The principle operating indices of reversible follow-up systems can be analyzed by using the general dynamics equation, derived from the original system of differential equations at (1).

The general dynamics equation for the operation of an RFS for the system studied has the form

(8)

Under steady-state operating conditions, when $P\alpha_o(P) = P\alpha_1(P)$ = constant

$$M_{\alpha_1}(P) = \frac{C_1(P)}{C_2(P)} [M_1(P) + b_2(P) P\alpha_1(P)] + b_1(P) P\alpha_o(P). \tag{9}$$

Solving this equation with respect to $P\alpha_o(P) = P\alpha_1(P) = P\alpha(P)$ we obtain

(10)

Eq. (10) establishes one of the basic operating indices for a reversible system, that of the sensitivity of the RFS to the action of external moments.

It is obvious that $P\alpha_o = P\alpha_1 = \text{const}$ is possible when one of the following conditions is satisfied

$$\begin{aligned} &M_o(P)\,C_2(P) - M_1(P)\,C_1(P) = \text{const};\\ &M_o(P) = \text{const}; \quad M_1(P) = \text{const}. \end{aligned} \tag{11}$$

The sensitivity of operation of a reversible follow-up system with respect to externally acting moments can be established for steady-state conditions from the following relationships.

When $M_o = \text{const}$; $M_1 = 0$; $P\alpha_o = P\alpha_1 = P\alpha = \text{const}$

$$\frac{P\alpha}{M_o} = \frac{C}{b_2C_1 + b_1C_2} = D_1. \tag{12}$$

When $M_1 = \text{const}$; $M_o = 0$; $P\alpha_o = P\alpha_1 = \text{const}$

$$\begin{aligned} &= 0;\ P\alpha_{on} = P\alpha_{H} = \text{const}\\ &\frac{P\alpha}{M_1} = \frac{C_1}{b_2C_1 + b_1C_2} = D_2, \end{aligned} \tag{13}$$

where b_1 and b_2 are the coefficients of the transfer functions $b_1(P)$; $b_2(P)$.

The coefficients D_1 and D_2 can be called the coefficients of the quality factor for the system with respect to the external moments, and they establish the sensitivity of operation of the RFS under steady-state conditions.

One of the basic features of reversible follow-up systems distinguishing them from conventional automatic regulating systems is the property of active reflection at the operator s shaft of load moments applied to the manipulation element. This property of RFS can be analyzed using Eq. (8), the general work dynamics equation. What follows from this equation is that the operator perceives the external load moments $[M_1(P) + I_1P^2\alpha_1(P)]$ with a definite reflection proportionality factor.

The magnitude of the force (moment) reflection proportionality factor is the most important parameter of a reversible follow-up system. The proportionality factor for a structurally-symmetrical RFS arrangement is

(14)

From the general dynamics equation it also follows that the operator, in addition to the external load moments, also perceives the so-called internal moments inherent in the reversible follow-up system proper

$$M_{in_1}(P) = I_1 P^2 \alpha_{o_1}(P) + b_1(P) P \alpha_{o_1}(P);$$
$$M_{in_2}(P) = I_2 P^2 \alpha_1(P) + b_2(P) P \alpha_1(P). \qquad (15)$$

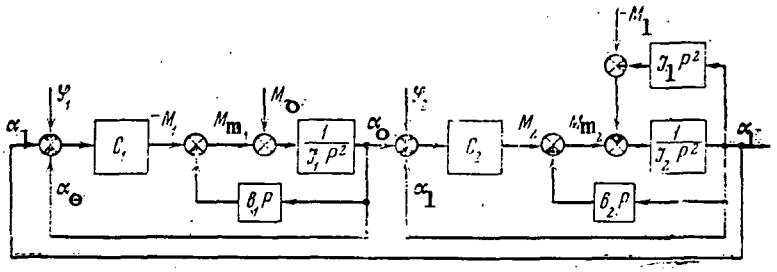

Figure 3.

The presence of "internal" moments causes an error in the operator's perception of the external load moments:

$$M_{o_1}(P) = n(P)\,[M_1(P) + I_1 P^2 \alpha_1(P)] + \Theta_M(P), \qquad (16)$$

where

$$\Theta_M(P) = M_{in_1}(P) + n(P)\,M_{in_2}(P) \qquad (17)$$

is the error in the operator's perception of the load moments.

It is obvious that precision in the operator's perception $[M_1(P) + I_1 P^2 \alpha_1(P)]$ is determined by system parameters, as well as by its mode of operation. Thus, if the friction moments in the system are negligibly small, $\Theta_M(P) = 0$, when the positions of the operator and load shafts are locked. Let us consider methods for compensating for internal moments in structurally symmetrical RFS arrangements.

The dynamics of the operation of an RFS with compensating signals $\varphi_1(P)$, and $\varphi_2(P)$, the block diagram of which is presented in Figure 3, can be described by the following system of differential equations written in Laplace transforms for zero initial conditions

$$
\begin{aligned}
M_{o_1}(P) &= M_1(P) + M_{in_1}(P);\\
M_2(P) &= M_1(P) + I_1 P^2 \alpha_1(P) + M_{in_2}(P);\\
M_1(P) &= C_1(P)[\alpha_{o_1}(P) - \alpha_1(P) - \varphi_1(P)];\\
M_2(P) &= C_2(P)[\alpha_{o_2}(P) - \alpha_1(P) - \varphi_2(P)].
\end{aligned}
\tag{18}
$$

The fundamental work dynamics equation for the structurally symmetrical RFS with compensating signals has the following form

(19)

Eq. (19) shows that two fundamentally different methods of compensating for "internal" moments are possible

I
$$
\varphi_1(P) - \varphi_2(P) = \frac{M_{in_1}(P)}{C_1(P)} + \frac{M_{in_2}(P)}{C_2(P)}; \tag{20}
$$

II
$$
\varphi_1(P) - \varphi_2(P) = m\left[\frac{M_{o_1}(P)}{C_1(P)} - \frac{M_1(P) + I_1 P^2 \alpha_1(P)}{C_2(P)}\right]. \tag{21}
$$

Here m is external moment signal transmission factor. /68

The first compensation method is based on feeding signals to the RFS that are proportional to the "internal" moments, and measured by indirect methods. Indirect methods of measuring M_{in} are extremely complex and imprecise, so the second method of compensating for M_{in} is most often used.

It should be noted that compensating signals proportional to the external influences on the operator, as well as on the load, shaft should be fed into the structurally symmetrical RFS.

If

$$
\varphi_1(P) - \varphi_2(P) = m\frac{M_{o_1}(P)}{C_1(P)},
$$

then

(22)

$$
M_{o_1}(P) = \frac{C_1(P)}{(m+1)C_2(P)}[M_{1_1}(P) + I_{1_1} P^2 \alpha_1(P) + M_{in_2}(P)] + \frac{M_{in_1}(P)}{m+1}.
$$

If

$$\varphi_1(P) - \varphi_2(P) = m \frac{M_1(P) + I_1 P^2 \alpha_1(P)}{C_2(P)},$$

then

$$M_{o1}(P) = \frac{C_1(P)(m+1)}{C_2(P)} [M_1(P) + I_1 P^2 \alpha_1(P)] + \frac{C_1(P)}{C_2(P)} M_{in_2}(P) + M_{in_1}(P). \quad (23)$$

As may be seen from Eqs. (22) and (23), signals proportional to the external influence of just one side change the proportionality factor for the operator's perception of load moments.

The simultaneous supply of signals proportional to the external influences attributable to operator and load, that is, the Eq. (21) condition is satisfied makes the general working dynamics equation for the RFS into

$$M_o(P) = \frac{C_1(P)}{C_2(P)} [M_1(P) + I_1 P^2 \alpha_1(P)] + \frac{[I_1 P^2 + b_1(P) P] \alpha_{o1}}{m+1} \quad (24)$$

As may be seen from Eq. (24), the influence of the "internal" moments on the precision of the operator's perception of load moments has been reduced by a factor of (m + 1).

Let us examine the operation of an RFS with couplings compensating for the "internal" moments under basic steady-state conditions.

When $P\alpha_o(P) = P\alpha_1(P) = P\alpha(P) = \text{const}$, the fundamental work dynamics equation for the RFS of Eq. (24) becomes

$$M_{o1}(P) = \frac{C_1(P)}{C_2(P)} M_1(P) + \frac{1}{m+1} \left[b_1(P) P\alpha_o(P) + \frac{C_1(P)}{C_2(P)} b_2(P) P\alpha_1(P) \right]. \quad (25)$$

Solving this equation for $P\alpha(P)$, we derive an expression which determines the sensitivity of operation of the RFS when compensating couplings are added

$$P\alpha(P) = (m+1) \frac{M_{o1}(P) C_2(P) - M_1(P) C_1(P)}{b_2(P) C_1(P) b_1(P) C_2(P)}. \quad (26)$$

The quality factors for the system in terms of external moments have the following form.

When M_o = const, M_1 = 0 /69

$$D_1 = \frac{P\alpha}{M_{op}} = (m+1)\frac{C_2}{b_2C_1 + b_1C_2}. \tag{27}$$

When M_1 = const: M_o = 0

$$D_2 = \frac{P\alpha}{M_1} = (m+1)\frac{C_1}{b_2C_1 + b_1C_2}. \tag{28}$$

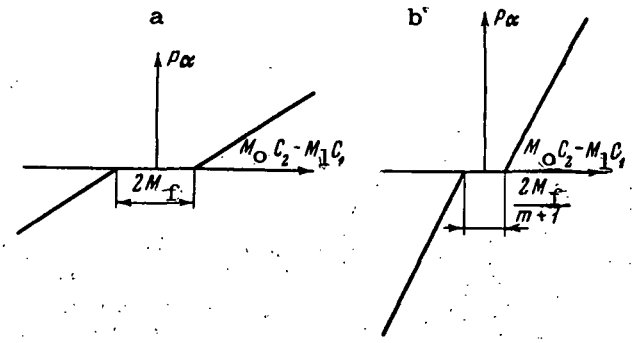

Figure 4. Graphs of the Pd(P) relationships. a - without compensating couplings b - with compensating couplings.

The system quality factor, that is, sensitivity to the action of external moments, increases by a factor of (m + 1).

Friction moments in the kinematic chains are included in the internal moments inherent in the reversible follow-up system. Dry friction moments, which considerably distort the functioning of the reversible system, can have very definite influence.

Let us consider the operation of an RFS with dry friction moments in the kinematic chains on the operator, as well as on the load side. The work dynamics equation for this system has the form

$$\begin{aligned} M_o(P) = \frac{C_1(P)}{C_2(P)}[M_1(P) + (I_1 + I_2)P^2\alpha_1(P) + b_2(P)P\alpha_1(P) + \\ + |M_{f_2}|\operatorname{sign} P\alpha_1(P)] + |M_{f_1}|\operatorname{sign} P\alpha_o(P) + \\ + [I_1P^2 + b_1(P)P]\alpha_o(P). \end{aligned} \tag{29}$$

It is obvious that the precision of the operator's perception of the load moments will be determined to a large extent by the magnitude of the dry friction moments.

The sensitivity of RFS operation is determined by the expression

$$P\alpha(P) = \frac{M_{o_2}(P)\,C_2(P) - M_{1_2}(P)\,C_1(P) - \{|M_{f_{1_2}}|\,C_2(P) + |M_{f_{2_2}}|\,C_1(P)\}}{b_2(P)\,C_1(P) + b_1(P)\,C_2(P)} \qquad (30)$$

when $$M_{o_2}(P)\,C_2(P) - M_{1_2}(P)\,C_1(P) > |M_{f_{1_2}}|\,C_2(P) + |M_{f_{2_2}}|\,C_1(P)$$

and $$P\alpha(P) = 0$$

when $$M_{o_2}(P)\,C_2(P) - M_{1_2}(P)\,C_1(P) < |M_{f_{1_2}}|\,C_2(P) + |M_{f_{2_2}}|\,C_1(P). \qquad (31)$$

Consequently, the graph of $P\alpha(P) = f[M_o C_2 - M_1 C_1]$ has a dead zone (Figure 4a).

With the supply to the structurally symmetrical arrangement of a compensating signal of the type

$$\varphi_1(P) - \varphi_2(P) = m\left[\cdot^{M}\right.$$

the work dynamics equation for the RFS can be written

70

$$\begin{aligned} M_{o_2}(P) = {} & \frac{C_1(P)}{C_2(P)}[M_1(P) + I_1 P^2 \alpha_{1_2}(P) + \\ & + \frac{C_1(P)}{C_2(P)(m+1)}\{[I_2 P^2 + b_2(P)\,P]\,\alpha_{1_2}(P) + |M_{f_{2_2}}|\,\mathrm{sign}\,P\alpha_{1_2}(P)\} + \\ & + \frac{1}{m+1}\{[I_1 P^2 + b_1(P)\,P]\,\alpha_{o_2}(P) + |M_{f_{1_2}}|\,\mathrm{sign}\,P\alpha_{o_2}(P)\}. \end{aligned} \qquad (32)$$

The sensitivity of the operation of an RFS with compensating couplings, signals for which are proportional to the external moments, is determined by the following expression when friction is present in the kinematic chains

$$P\alpha(P) = (m+1)\frac{M_{o_2}(P)\,C_2(P) - M_{1_2}(P)\,C_1(P)}{b_2(P)\,C_1(P) + b_1(P)\,C_2(P)} \qquad (33)$$

when

If this condition is not satisfied, $P\alpha(P) = 0$. Figure 4 shows the relationship

when compensating couplings proportional to the external influences are added.

Thus, the presence of friction moments in the kinematic chains of an RFS reduces its sensitivity to the action of external moments and reduces the precision with which the operator perceives load moments. The insertion of compensating couplings in the structurally symmetrical RFS very nearly balances this negative influence.

Let us consider the principal parameters and operating indices for reversible follow-up systems of the structurally asymmetrical type. Figure 5 is a block diagram of such a system, and the dynamics of its operation can be described by the following system of differential equations written in Laplace transforms for zero initial conditions

$$\begin{aligned}
&M_{m_1}(P) = I_1 P^2 \alpha_0(P) - M_0(P);\\
&M_{m_2}(P) = (I_2 + I_1) P^2 \alpha_1(P) + M_1(P);\\
&M_{m_1}(P) = C_1(P)\{M_0(P) - I_{K_1} P^2 \alpha_{0p} - B_1(P) P\alpha_0(P) -\\
&\qquad - n[M_1(P) + (I_1 + I_{K_2}) P^2 \alpha_1(P)]\};\\
&M_{m_2}(P) = C_2(P)[\alpha_0(P) - \alpha_1(P)] - b_2(P) P\alpha_1(P),
\end{aligned} \tag{34}$$

where

$$I_1 = I_{m_1} + I_{K_1}; \qquad I_2 = I_{m_2} + I_{K_2}.$$

Note that in this arrangement the moment sensors measure the external moments on the master or slave side, as well as the dynamic moments attributable to the inertia in the links in the kinematic chain. This is explained by the fact that in practice it is extremely difficult to install the moment sensors in the output of the kinematic chain of the control or manipulation elements, so they are installed in the slave units after the motors.

We also noted that the magnitude of the moment of inertia of the kinematic chain is variable, and depends on its position.

This system of equations can be transformed into one more convenient for subsequent analysis 71

(35)

C2

$$W_{1m}(p) = \frac{n_1}{M_1} = \frac{k_{1m}}{(T_m p + 1)};$$

$$\Theta_M(P) = M_{in_1}(P) = n(P) M_{in_2}(P) = (I_{K_1} + I_{m_1}) P^2 \alpha_0(P) +$$
$$+ b_1(P) P \alpha_0(P) + n(P) [(I_{K_2} + I_{m_2}) P^2 \alpha_1(P) + b_2(P) P \alpha_1(P).$$

Figure 5.

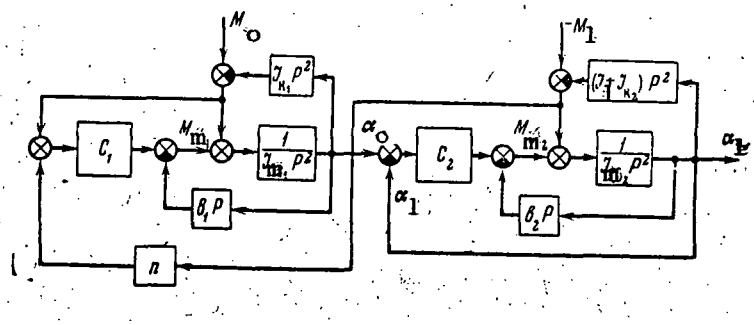

Figure 6.

Figure 6 is the structural diagram of a structurally asymmetrical reversible follow-up system. The general work dynamics equation for this system can be found by solving (35), and has the form

(36)

Since $C_1(P)$ is a transfer function with an extremely large amplification factor, we can consider that

$$M_0(P) \approx n[M_1(P) + (I_1 + I_{K_2}) P^2 \alpha_1(P)] +$$
$$+ I_{K_1} P^2 \alpha_0(P) + \frac{I_{m_1} P^2 \alpha_0(P) + b_1(P) P \alpha_0(P)}{C_1(P)} \qquad (37)$$

or

(38)

where

n is moment proportionality factor;

$\Theta_M(P)$ is the error in the operator's perception of the load moments

(39)

At the same time, we have the following for a structurally symmetrical scheme without compensation for "internal" moments /72

Consequently, the structurally asymmetrical RFS scheme considered provides sufficiently complete compensation for that part of the "internal" moments inside the circuit of the arrangement used for moment measurers.

The general work dynamics equation for a structurally asymmetrical reversible follow-up system under steady-state conditions when $P\alpha_o(P) = P\alpha_1 = (P)\alpha(P) =$ = const has the form

$$M_{o\gamma}(P) \tag{40}$$

In accordance with this equation, the sensitivity of operation of a structurally asymmetrical RFS can be expressed by the following relationship

(41)

The RFS quality factors in terms of the external moments can be determined under steady-state operating conditions.

When

$$M_{o} = \text{const} \qquad M_1 = 0$$

When

$$M_1 = \text{const} \qquad M_{o} = 0 \qquad \frac{P}{M}$$

The conclusion is that under steady-state operating conditions the precision of the operator's perception of the load moments, as well as the sensitivity of the structurally asymmetrical RFS, is determined only by the loader parameters, that is, by the parameters for the active force reflection channel. It is

obvious that the accuracy of position tracking by such a system is determined only by the parameters of a slave drive closed with respect to position.

The structurally asymmetrical RFS scheme does not require the use of additional couplings to compensate for internal moments. All this means that in designing master-slave manipulator control systems we should use structurally asymmetrical schemes which permit delimiting the functions of the direct and return channels while ensuring specified quality indices.

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